

The Optimal Timing of Transition to New Environmental Technology in Economic Growth

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Abstract: This study aims at highlighting the choice of timing for technological change vis-à-vis environmental quality in economic development. We develop a model that addresses the transition of environmental technology from old one to new one. Findings obtained are innovative in that they depict when as well as how transition of environmental technology occurs. It is shown that the timing is endogenous and is characterized by the properties of the economy: in particular, the timing is dependent upon whether the economy is developing or developed.

Keywords: Environmental quality, Technological change, Optimal stopping.

JEL Codes: O13, O31, Q55.

1. Introduction

Since 1970s, many researchers have worked on resource and environmental issues in light of economic growth. Classical studies that incorporated exhaustible resources into the Ramsey model include Dasgpta and Heal (1974), Stiglitz (1974), Solow (1974), etc. These studies pioneered theoretical exploration to the possible economic growth with natural resource constraints.

In 1980s, the environment began to be recognized as another important factor that determines the trajectories of growth. At the same time, the theory of economic growth began to change: “endogenous change” comes to a central issue in the economic literature: Romer (1990), Lucas (1988), etc., established “endogenous economic growth theory.” These two streams converge to a bunch of studies after late 1990s, addressing that endogenous technological change plays central roles in sustainable development.

Barbier (1999) and Tahvonen and Salo (2001) developed their endogenous growth models combined with resource constraints, highlighting the critical role of technology policy. On the other hand, Bovenberg and Smulders (1995), Schou (2000), and Groth and Shou (2002) introduced “knowledge” and/or “know-how” into their model frames of production sectors, and discussed that these accumulation would contribute to mitigating environmental constraints.

Cunha-e-Sá and Reis (2007) contributed to the literature in particular in that they casted light on the optimal timing of adoption of new environmental technologies: as the economy grows, new pollution-abating technology becomes indispensable; it requires investment efforts at the time of deployment; the optimal timing will be determined in an endogenous manner. In fact, they incorporated “environmental quality” and “clean technology” into a typical Ramsey-type model. They assumed that the level of clean technology could change discontinuously once “grade-up” happened. Grade-up of the technology may require costs such as depreciation of capital, investment for research and development, etc. Thus, consideration for the net benefit determines the optimal timing of grade-up.

While Cunha-e-Sá and Reis made a point by underlining the significance of “the optimal timing” in the framework of the environment and economic growth, some debatable issues remain.

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The most debatable issue is that technological change in clean technology is represented by jump of the level. Allowing such a sudden change of the level implies that the change does not require accumulation of some stocks; that is, the change is not due to stocks, but is due to temporal flows. More specifically, the nature of technological change in clean technology in Cunha-e-Sá and Reis has nothing to do with any stocks of capital, knowledge, know-how, human capital, etc., and thus cannot be endogenously described. This nature contrasts to the fundamental idea of modern endogenous growth theory: in the theory, one of key drivers for economic growth is considered to be intellectual capital. In this respect, their treatment of technological change for clean technology seems old-fashioned. It should be the rate of change in technology level, rather than change itself, that is subject to jumps.

Another important issue is their treatment of investment cost. In their model, required cost for technological change is represented as a fixed cost the amount of which is proportional to the difference between old and new clean technology levels. Also, it is taken into account as a part of objective function. This setting seems strange: as long as we assume a closed economy, such investment cost must be paid somebody in the economy, meaning that the objective function should not explicitly include the cost; the investment cost should be treated as a decline in consumption or that in asset holdings. Cunha-e-Sá and Reis do not seem to provide any explanation on this point.

Including investment cost in the objective function causes another problem. In Ramsey model, the objective function is basically the sum of temporal utilities that are discounted to the present. It does not have any unit. If we want to subtract investment costs from it, the unit of the cost—usually in monetary terms—must be transformed somehow. The coefficient for the transformation should be interpreted as a shadow price of the clean technology investment. Thus, it must naturally have a dynamics and be determined endogenously. However, in Cunha-e-Sá and Reis it is treated as a fixed parameter while they do not provide any clear explanation on this point.

Looking into these issues that are left in the study of Cunha-e-Sá and Reis, we find a possible direction for further research: we would modify their model by focusing upon an abrupt change in the rate of clean technology improvement rather than that in the level itself of clean technology, and also by working on a traditional objective function that simply consists of the sum of discounted temporal utilities. It is our purpose that we explore such a direction.

This study aims at highlighting the choice of timing for technological change vis-à-vis environmental quality in economic development. Based on the spirit and mathematical treatment of Cunha-e-Sá and Reis, we develop an analytical model that addresses the optimal timing of transition of environmental technology from old one to new one.

The paper is organized as follows: Section 2 describes the model. Section 3 investigates the dynamics of the model by the use of dynamic programming and maximum principle techniques. Section 4 proposes three propositions and discusses their implications. Section 5 concludes.

2. The Model

The model is basically a Ramsey-type model. The economy is closed, i.e. no international trade is assumed. The population is assumed to be constant. Let us define notations as follows:

- Y : Production output
- K : Capital
- C : Consumption
- Q : Environmental quality
- α : Environmental technology level

A representative economic agent represents the household whose time-additive utility is determined by not only consumption (C) but also “environmental quality” (Q). More specifically, we define:

$$U_t = \frac{(C_t Q_t^\mu)^{1-\sigma} - 1}{1-\sigma}, \quad 0 < \mu < 1, \quad \sigma > 0, \quad \sigma \neq 1 \quad (2.1)$$

where σ represents the magnitude of the elasticity of marginal utility with respect to $C_t Q_t^\mu$.

Note that subscript t indicates time, which is typically years. This applies in all the remainder.

We impose the following restrictions:

$$\mu(1-\sigma) < 1, \quad \mu(1-\sigma) < \sigma.$$

These restrictions determine signs of partial derivatives of the utility function as follows:

$$\partial U / \partial C > 0, \quad \partial^2 U / \partial C^2 < 0, \quad \partial U / \partial Q > 0, \quad \text{and} \quad \partial^2 U / \partial Q^2 > 0.$$

The cross second derivative is:

$$\frac{\partial^2 U}{\partial C \partial Q} = \mu(1-\sigma)C^{-\sigma}Q^{\{\mu(1-\sigma)-1\}},$$

the sign of which remains undetermined. In fact, the sign depends on the magnitude of the elasticity of marginal utility (σ). We will discuss this point later.

Production sector is described rather in a simple manner: we assume that output is proportional to capital input, that is:

$$Y_t = AK_t. \quad (2.2)$$

With this production function, capital accumulation is subject to the following dynamics.

$$dK_t/dt = AK_t - C_t \quad (2.3)$$

We now introduce the roles of environmental quality and technology: we assume that the environmental quality (Q) is a function of the consumption (C) as well as “environmental technology level” (a). The relationship is assumed as follows:

$$Q_t = a_t C_t^{-\alpha}, \quad \alpha > 0 \quad (2.4)$$

This functional form indicates that environmental quality declines as consumption grows, but that it may improve as environmental technology progresses.

The progress of the environmental technology level itself may occur as the economy grows. Considering capital stock (K) as a proxy of economic growth, we assume that the growth rates of a and K are proportional to each other:

$$\dot{a}_t/a_t \propto \dot{K}_t/K_t$$

Introducing a positive coefficient η , the relation is written as follows:

$$\dot{a}_t/a_t = \eta \dot{K}_t/K_t$$

Notice that the coefficient determines the rate of environmental technology progress together with that of capital accumulation. Thus, if the economy has an internal mechanism that changes the coefficient, the mechanism accelerates the technological progress. The change in the coefficient does not necessarily gradual. Rather, it may be discontinuous, and may happen in an innovative manner. We call this type of discontinuous change in the rate of environmental technology progress “transition.” Since it is a result of some innovation, we may assume that it may happen at one time in the infinite time horizon with a huge capital investment. The investment includes not only investing money in a usual way, but also scrapping existing physical capitals because part of physical capitals may become obsolete and old-fashioned once such an innovation occurs.

Let T denote the time of transition. For the purpose of mathematical treatment, let us introduce a notation T_- such that:

$$T_- \equiv \lim_{t \rightarrow T^-, t < T} t.$$

Transition of environmental technology is represented by an abrupt change of the coefficient that links two growth rates of technology and capital stock to each other. More specifically, the following relation holds:

$$\frac{\dot{a}_t/a_t}{\dot{K}_t/K_t} = \begin{cases} \eta_0 & \text{for } t \leq T_- \\ \eta_1 & \text{for } t \geq T \end{cases} \quad (2.5)$$

$$0 \leq \eta_0 < \eta_1$$

The relation indicates that the environmental technology level (a) is written as a function of capital stock (K) as follows:

$$a_t = \begin{cases} \bar{a} K_t^{\eta_0} & \text{for } t \leq T_- \\ \bar{a} K_t^{\eta_1} & \text{for } t \geq T \end{cases} \quad (2.6)$$

Let K_{T-} denote the capital stock at the time of T_- , that is:

$$K_{T-} \equiv \lim_{t \rightarrow T^-, t < T} K_t$$

As is discussed above, the transition is a result of one-time capital investment decision that includes scrapping of existing physical capital. The investment is thus described as substantial decline in the present capital. More specifically, we assume that following relationship holds:

$$K_T = \beta K_{T-}, \quad 0 < \beta \leq 1. \quad (2.7)$$

The coefficient β represents the rate of depreciation of existing physical capital at the time of transition.

Note that in (2.6), $\bar{a}$ represents an absolute level of the technology. While we need to assume a constant value somehow for it as a modeling frame, the absolute level itself does not play any important role in the model. Only changes in the level do play the role. Thus, the value of $\bar{a}$ itself is not important and may remain undetermined. Instead, we introduce the following restriction:

$$K_{T-}^{\eta_0} \equiv K_T^{\eta_h} \quad (2.8)$$

This restriction indicates continuity of the technology level at the very moment of transition T . We assume that $\bar{a}$ is fixed *ex post* so that (2.8) holds.

Given the above setting, the representative economic agent chooses the consumption path as well as the time of transition. While the decision for the consumption path is a typical optimal control, the decision for the choice of the time of transition constitutes an optimal stopping problem. The economy follows the optimal solution of the following dynamic programming problem.

$$\begin{aligned} J(K_0) = & \text{Max}_{T, \{C_t\}} \int_0^{T-} U_t e^{-\rho t} dt + \int_T^{\infty} U_t e^{-\rho t} dt \\ \text{s.t.} \quad & U_t = \frac{(C_t Q_t^\mu)^{1-\sigma}}{1-\sigma}, \quad 0 < \mu < 1, \quad \sigma > 0, \quad \sigma \neq 1 \\ & dK_t/dt = AK_t - C_t \\ & K_T = \beta K_{T-}, \quad 0 < \beta \leq 1 \\ & Q_t = a_t C_t^{-\alpha}, \quad \alpha > 0 \\ & a_t = \begin{cases} \bar{a} K_t^{\eta_0} & \text{for } t \leq T- \\ \bar{a} K_t^{\eta_h} & \text{for } t \geq T \end{cases} \end{aligned} \quad (2.9)$$

The problem has control variables $\{C_t\}_{t \in [0, \infty]}$ and T and a state variable $\{K_t\}_{t \in [0, \infty]}$. Therefore, we can solve the problem by the following procedure:

First step: optimizing the value function for a given capital stock for after-transition.

$$\varphi(K_T) = \text{Max}_{\{C_t\}} \int_T^{\infty} U_t e^{-\rho(t-T)} dt \quad (2.10)$$

Second step: optimizing the value function for a given transition time for the whole time horizon.

$$V(T) = \text{Max}_{\{C_t\}} \int_0^{T-} U_t e^{-\rho t} dt + \varphi(K_T) e^{-\rho T} \quad (2.11)$$

Final step: maximizing (2.11) with respect to T .

$$J(K_0) = \text{Max}_T V(T) \quad (2.12)$$

The next section investigates the details of the procedure.

3. Analysis

Suppose that technology transition has already occurred at time T . Capital stock at the time is fixed at the level of K_T . The optimization problem thereafter is described as follows:

$$\begin{aligned} \varphi(K_T) = & \text{Max}_{\{C_t\}} \int_T^{\infty} \frac{(C_t Q_t^\mu)^{1-\sigma}}{1-\sigma} e^{-\rho(t-T)} dt \\ \text{s.t.} \quad & dK_t/dt = AK_t - C_t \\ & Q_t = a_t C_t^{-\alpha} \\ & a_t = \bar{a} K_t^{\eta_h} \end{aligned} \quad (3.1)$$

K_T is given; $t \geq T$.

To solve the problem, we introduce the current-valued Hamiltonian as follows:

$$\mathcal{H} = \frac{\left\{ C_t \left(\bar{a} K_t^\eta C_t^{-\alpha} \right)^\mu \right\}^{1-\sigma} - 1}{1-\sigma} + \lambda_t (AK_t - C_t) \quad (3.2)$$

Note that λ_t represents a current-valued shadow price of capital.

First order necessary conditions for the optimality are written as follows:

$$\partial \mathcal{H} / \partial C = (1 - \alpha\mu) \bar{a}^\mu K_t^{\eta\mu(1-\sigma)} C_t^{(1-\alpha\mu)(1-\sigma)} C_t^{-1} - \lambda_t = 0 \quad (3.3)$$

$$\partial \mathcal{H} / \partial K = \eta_1 \mu \bar{a}^\mu K_t^{\eta\mu(1-\sigma)} C_t^{(1-\alpha\mu)(1-\sigma)} K_t^{-1} + \lambda_t A = \rho \lambda_t - \dot{\lambda}_t \quad (3.4)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t K_t = 0 \quad (3.5)$$

Combining (3.3) and (3.4), we have the following differential equation with respect to C_t/K_t :

$$\frac{\partial \left(\frac{C_t}{K_t} \right)}{\partial t} = M_1 \left(\frac{C_t}{K_t} \right)^2 - \Lambda_1 \frac{C_t}{K_t}, \quad t \geq T \quad (3.6)$$

where

$$\begin{cases} M_1 = \frac{\eta_1 \mu + 1 - \alpha\mu}{1 - \alpha\mu} > 0 \\ \Lambda_1 = \frac{\rho - A(1-\sigma)(\eta_1 \mu + 1 - \alpha\mu)}{1 - (1 - \alpha\mu)(1-\sigma)} > 0 \end{cases}$$

Solving (3.6) with the condition (3.5), we obtain the solution as follows:

$$\begin{aligned} \frac{C_t}{K_t} &= \frac{(1 - \alpha\mu) [\rho - A(1-\sigma)(\eta_1 \mu + 1 - \alpha\mu)]}{[1 - (1 - \alpha\mu)(1-\sigma)](\eta_1 \mu + 1 - \alpha\mu)} \\ &= \frac{\Lambda_1}{M_1} > 0, \quad t \geq T \end{aligned} \quad (3.7)$$

The above solution shows that the proportion of consumption to capital stock remains constant after the technology transition. This rule suggests that consumption and capital stock grow at the same rate after the transition. In fact, their growth rates are found as follows:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{K}_t}{K_t} = \frac{A\eta_1 \mu + (1 - \alpha\mu)(A - \rho)}{[1 - (1 - \alpha\mu)(1-\sigma)](\eta_1 \mu + 1 - \alpha\mu)}, \quad t \geq T \quad (3.8)$$

Employing (3.7) and (3.8), we obtain the integral of (3.1) as follows:

$$\varphi(K_T) = \frac{\bar{a}^{\mu(1-\sigma)} \left(\frac{\Lambda_1}{M_1} \right)^{(1-\alpha\mu)(1-\sigma)} K_T^{(1-\sigma)(\eta_1 \mu + 1 - \alpha\mu)}}{(1-\sigma)\Lambda_1} - \frac{1}{\rho(1-\sigma)} \quad (3.9)$$

Having obtained $\varphi(K_T)$ as (3.9), we are ready to work on (2.11). Let us rewrite the problem:

$$V(T) = \max_{\{C_t\}} \int_0^{T-} \frac{(C_t Q_t^\mu)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt + \varphi(K_T) e^{-\rho T} \quad (3.10)$$

$$\begin{aligned} \text{s.t.} \quad & dK_t/dt = AK_t - C_t \\ & K_T = \beta K_{T-}, \quad 0 < \beta \leq 1 \\ & Q_t = a_t C_t^{-\alpha} \\ & a_t = \bar{a} K_t^{\eta_0} \\ & T \text{ is given; } t \leq T_- \end{aligned}$$

Again, we introduce the current-valued Hamiltonian, and derive first order necessary conditions as follows:

$$\partial \mathcal{H} / \partial C = (1 - \alpha\mu) \bar{a}^\mu K_t^{\eta_0 \mu(1-\sigma)} C_t^{(1-\alpha\mu)(1-\sigma)} C_t^{-1} - \lambda_t = 0 \quad (3.11)$$

$$\partial \mathcal{H} / \partial K = \eta_0 \mu \bar{a}^\mu K_t^{\eta_0 \mu(1-\sigma)} C_t^{(1-\alpha\mu)(1-\sigma)} K_t^{-1} + \lambda_t A = \rho \lambda_t - \dot{\lambda}_t \quad (3.12)$$

$$\lambda_{T-} = \frac{\partial \varphi(K_T)}{\partial K_T} \cdot \frac{\partial K_T}{\partial K_{T-}} \quad (3.13)$$

Because capital stock changes discontinuously at time T , so does its shadow price. It is easily shown that:

$$\lambda_{T-} = \lambda_T \cdot \beta. \quad (3.14)$$

Notice that (3.11) and (3.12) are similar to (3.3) and (3.4), respectively. The only difference is that we replaced η_t to η_0 . Thus, they provides a similar differential equation with (3.6):

$$\frac{\partial \left(\frac{C_t}{K_t} \right)}{\partial t} = M_0 \left(\frac{C_t}{K_t} \right)^2 - \Lambda_0 \frac{C_t}{K_t}, \quad t \leq T_- \quad (3.15)$$

where

$$\begin{cases} M_0 = \frac{\eta_0 \mu + 1 - \alpha \mu}{1 - \alpha \mu} > 0 \\ \Lambda_0 = \frac{\rho - A(1 - \sigma)(\eta_0 \mu + 1 - \alpha \mu)}{1 - (1 - \alpha \mu)(1 - \sigma)} > 0 \end{cases}$$

Due to (2.7), (2.8) and (3.14), the terminal value of (3.15) must satisfy the following:

$$\frac{C_{T-}}{K_{T-}} = \frac{C_T}{K_T} \beta^{-\frac{(1-\alpha\mu)(1-\sigma)}{1-(1-\alpha\mu)(1-\sigma)}} \quad (3.16)$$

Solving (3.15) with its terminal value (3.16), we obtain the following dynamics:

$$\frac{C_t}{K_t} = \frac{\Lambda_0/M_0}{1 + e^{\Lambda_0(t-T)}(\delta - 1)}, \quad t \leq T_- \quad (3.17)$$

where

$$\delta = \frac{(\eta_h \mu + 1 - \alpha \mu) \left\{ \rho - A(1 - \sigma)(\eta_0 \mu + 1 - \alpha \mu) \right\}}{(\eta_0 \mu + 1 - \alpha \mu) \left\{ \rho - A(1 - \sigma)(\eta_h \mu + 1 - \alpha \mu) \right\}} \beta^{\frac{(1-\alpha\mu)(1-\sigma)}{1-(1-\alpha\mu)(1-\sigma)}}$$

From (3.17), we obtain the dynamics that capital stock must follow before the transition happens, as follows:

$$K_t = K_0 \cdot e^{At} \left[\frac{1 + e^{-\Lambda_0(T-t)}(\delta - 1)}{e^{\Lambda_0 t} + e^{-\Lambda_0(T-t)}(\delta - 1)} \right]^{\frac{1}{M_0}}, \quad t \leq T_- \quad (3.18)$$

This enables us to evaluate $V(T)$ in (3.10), which is a task in the next section.

4. Propositions

Our final step is to maximize $V(T)$ of (3.10) with respect to T , given (2.7), (3.9) and (3.18). The solution would provide the optimal time of transition T^* . Although solving for it analytically is not easy task, examining the properties of the solution is rather easy. This is done by taking the derivative of $V(T)$ with respect to T . From (3.10), we obtain the following:

$$\frac{dV(T)}{dT} = \frac{\bar{a}^{\mu(1-\sigma)}}{1 - \sigma} e^{-\rho T} \left\{ \left(\frac{\Lambda_0}{M_0} \right) \frac{1}{\delta} \right\}^{(1-\alpha\mu)(1-\sigma)} K_{T-}^{(1-\sigma)(\eta_0 \mu + 1 - \alpha \mu)} \left\{ 1 - K_{T-}^{(1-\sigma)(\eta_h - \eta_0) \mu} \beta^{\frac{(1-\alpha\mu)(1-\sigma)}{1-(1-\alpha\mu)(1-\sigma)} + (1-\sigma)\eta_h \mu} \right\} \quad (4.1)$$

To have a finite T^* , we must have the following conditions:

$$\begin{aligned} dV(T)/dT \Big|_{T=T^*} &= 0 \quad \text{if } 0 < T^* < \infty \\ dV(T)/dT \Big|_{T=T^*} &< 0 \quad \text{if } T^* = 0 \end{aligned}$$

These conditions lead to the following proposition.

Proposition 1.

For a finite T^* to exist, that is, $T^* < \infty$, the following condition is necessary:

$$(1 - \sigma)(A - \Lambda_0/M_0) \geq 0 \quad (4.2)$$

Conversely, if $(1 - \sigma)(A - \Lambda_0/M_0) < 0$ holds, then the transition of environmental technology never occurs (i.e. $T^* = \infty$).

(Proof is omitted.)

Let us examine the implication of the proposition. The necessary condition (4.2) is equivalent to the following:

$$\begin{aligned} &\text{Either } 1 > \sigma \text{ and } A \geq \Lambda_0/M_0 \\ &\text{or } 1 < \sigma \text{ and } A \leq \Lambda_0/M_0 \text{ hold.} \end{aligned}$$

From (3.17), we have:

$$\left(\frac{C_{T-}}{K_{T-}} \right) = \lim_{t \rightarrow T-} \frac{\Lambda_0/M_0}{1 + e^{\Lambda_0(t-T)}(\delta - 1)},$$

which implies:

$$\Lambda_0/M_0 = (C_{T-}/K_{T-})\delta. \quad (3.17')$$

Thus, we have:

$$A \begin{cases} \geq \\ \leq \end{cases} \Lambda_0/M_0 \text{ if and only if } 1/\delta \begin{cases} \geq \\ \leq \end{cases} C_{T-}/AK_{T-} = C_{T-}/Y_{T-}.$$

Notice that C_{T-}/Y_{T-} represents average consumption propensity just before the transition. Then, we can further translate the condition (4.2) into the following conditions:

- If $1 > \sigma$ holds, average consumption propensity just before the transition is less than or equal to $1/\delta$.
- If $1 < \sigma$ holds, average consumption propensity just before the transition is larger than or equal to $1/\delta$.

As was shown in Section 2, σ represents the magnitude of the elasticity of marginal utility and determines the sign of the cross second derivative of the utility function as follows:

$$\frac{\partial^2 U}{\partial C \partial Q} = \mu(1 - \sigma)C^{-\sigma}Q^{\{\mu(1-\sigma)-1\}}$$

When $1 > \sigma$ holds, marginal utility of environmental quality ($\partial U/\partial Q$) turns out to be increasing in consumption. This means that a higher consumption would yield a higher marginal utility of environmental quality, implying that the potential of utility improvement with respect to environmental quality improvement is higher when consumption is high than otherwise. Similar interpretation can be possible by replacing C and Q to each other: marginal utility of consumption ($\partial U/\partial C$) turns out to be increasing in environmental quality, meaning that the potential of utility improvement with respect to increase in consumption becomes higher as environmental quality improves. In short, combining consumption increase with environmental quality improvement always improves utility, which means that consumption and environmental quality are complimentary goods to each other. The society prefers more environmental quality as the consumption grows.

When $1 < \sigma$ holds, on the other hand, marginal utility of environmental quality ($\partial U/\partial Q$) turns out to be decreasing in consumption. This means that a higher consumption would yield a lower marginal utility of environmental quality. Again, replaying C and Q to each other, we can say that a higher environmental quality would yield a lower marginal utility of consumption. In this case, combining decline in consumption with environmental quality improvement is desirable. This situation indicates that consumption and environmental quality are substitute goods to each other. The society prefer less environmental quality as the consumption grows.

Intuitively speaking, the society with the condition of $1 > \sigma$ would care about the environment while that with the condition of $1 < \sigma$ would not.

Combining these interpretations for Λ_0/M_0 and σ together, Proposition 1 is interpreted in words as follows:

Interpretation of Proposition 1

For the transition to occur in a finite time horizon, the economy must be in either of the following two situations:

- The economy highly prefers environmental quality, and its current average consumption propensity is lower than a certain value ($1/\delta$).

- The economy rather prefers consumption to environmental quality, and its current average consumption propensity is higher than a certain value ($1/\delta$).

Interpreting Proposition 1 in this way, we may make a conjecture about how the transition occurs. For the former case of the above, a lower consumption propensity indicates a higher saving rate. This implies that the economy wants to make investment to develop capital stock. That is to say, the economy is in the phase of capital accumulation. Conversely, in the later case of the above, the economy is depreciating its capital stock. Further more, while transition may occur in the course of capital accumulation in the former case, it may occur with capital depreciation in the latter case.

This conjecture is in fact justified by the following proposition.

Proposition 2.

Suppose that $(1-\sigma)(A-\Lambda_0/M_0) > 0$ holds. If the following conditions:

$$\text{either } 1 > \sigma \text{ and } K_0 < \left(\beta^{-1}\right)^{\left\{\frac{(1-\alpha\mu)}{1-(1-\alpha\mu)(1-\sigma)}+\eta_h\mu\right\}/\left\{(\eta_h-\eta_0)\mu\right\}} \quad (4.3a)$$

$$\text{or } 1 < \sigma \text{ and } K_0 > \left(\beta^{-1}\right)^{\left\{\frac{(1-\alpha\mu)}{1-(1-\alpha\mu)(1-\sigma)}+\eta_h\mu\right\}/\left\{(\eta_h-\eta_0)\mu\right\}} \quad (4.3b)$$

hold, then there exists an optimal transition time T^* such that $0 < T^* < \infty$, and it is obtained by solving the following equation:

$$K_0 \cdot \delta^{\frac{1}{M_0}} \frac{e^{AT^*}}{\left[e^{\Lambda_0 T^*} + \delta - 1\right]^{\frac{1}{M_0}}} = \left(\beta^{-1}\right)^{\left\{\frac{(1-\alpha\mu)}{1-(1-\alpha\mu)(1-\sigma)}+\eta_h\mu\right\}/\left\{(\eta_h-\eta_0)\mu\right\}}. \quad (4.4)$$

Moreover, the capital stock at the time is provided as follows:

$$K_{T^*}^* = \left(\beta^{-1}\right)^{\left\{\frac{(1-\alpha\mu)}{1-(1-\alpha\mu)(1-\sigma)}+\eta_h\mu\right\}/\left\{(\eta_h-\eta_0)\mu\right\}} \quad (4.5)$$

(Proof is omitted.)

Having known (4.5), we can rewrite conditions (4.3a) and (4.3b) as follows:

$$\text{if } 1 > \sigma, \quad K_0 < K_{T^*}^* \text{ must hold, and} \quad (4.6a)$$

$$\text{if } 1 < \sigma, \quad K_0 > K_{T^*}^* \text{ must hold.} \quad (4.6b)$$

Similarly, (4.4) is equivalent to the following:

$$\delta^{\frac{1}{M_0}} \frac{e^{AT^*}}{\left[e^{\Lambda_0 T^*} + \delta - 1\right]^{\frac{1}{M_0}}} = \frac{K_{T^*}^*}{K_0} \quad (4.7)$$

This equation indicates that for sufficiently large T^* , the following approximation holds.

$$\delta^{\frac{1}{M_0}} e^{(A-\Lambda_0/M_0)T^*} \approx K_{T^*}^* / K_0$$

In addition, if $\delta \approx 1$ holds, then a further approximation is obtained:

$$e^{(A-\Lambda_0/M_0)T^*} \approx K_{T^*}^* / K_0$$

This approximation shows that:

$$\text{for } K_0 < K_{T^*}^*, \quad e^{(A-\Lambda_0/M_0)T^*} > 1, \quad \text{i.e. } A > \Lambda_0/M_0, \text{ and}$$

$$\text{for } K_0 > K_{T^*}^*, \quad e^{(A-\Lambda_0/M_0)T^*} < 1, \quad \text{i.e. } A < \Lambda_0/M_0.$$

However, we have already assumed that $(1-\sigma)(A-\Lambda_0/M_0) > 0$ as a sufficient condition for the existence of a finite T^* . Thus, we find the above conditions are equivalent to (4.6a) and (4.6b). Then, we finally realize that (4.7) is approximately consistent to (4.6a) and (4.6b).

Another interesting finding is the independency of $K_{T^*}^*$ from the initial capital stock K_0 .

In fact, (4.5) indicates that $K_{T^*}^*$ is a constant value and does not depend upon K_0 . This means that no matter from which level the economy starts growing (or shrinking), transition of environmental technology occurs at the same capital stock level. That is to say, no matter the economy is developed

or developing, the transition occurs at the same economic conditions.

Our next attention, then, naturally comes to how the economy reaches the fixed capital stock level. We can find the answer from (4.6a) and (4.6b): from these conditions, we can consider two types of dynamic behavior.

- In case of $1 > \sigma$, capital stock K_t increases and hits the fixed level $K_{T^*}^*$ from below.
- In case of $1 < \sigma$, capital stock K_t declines and hits the fixed level $K_{T^*}^*$ from above.

These are depicted as Figures 1 and 2.

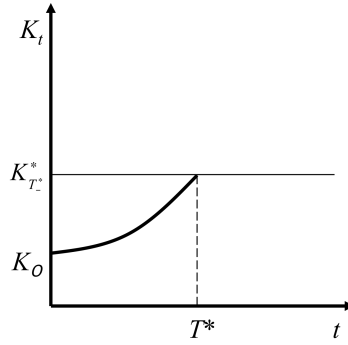


Figure 1. Capital path for the case of $1 > \sigma$

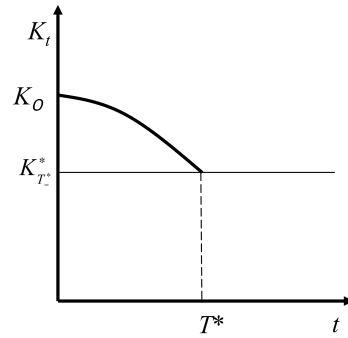


Figure 2. Capital path for the case of $1 < \sigma$

When $1 > \sigma$ holds (Figure 1), capital accumulation is rather typical: it gradually increases as time goes. On the other hand, when $1 < \sigma$ holds (Figure 2), capital stock declines, which does not seem a typical growth. For the economy to be able to take such a path, it must have already developed enough. In fact, the economy holds a huge capital, and is only consuming the capital. Such an economy must have once developed a large empire on the earth. A typical example may include the Holy Roman Empire, the British Empire, etc., in old days.

We have already discussed the meaning of σ in the above: whether σ is less or greater than the unity divides the economy in which consumption and environmental quality are complements to each other from the one in which they are substitutes. In the former one, as consumption grows, more environmental quality is needed. This motivates innovation for the technology. On the other hand, in the latter one, declining consumption motives innovation. The reason that consumption is declining is the economy has kept eating out its own assets. To recover the decline in the utility, environmental quality improvement becomes necessary.

Finally, let us examine the sensitivities of the solution with respect to parameters that determine the evolution of environmental technology. The following proposition is obtained:

Proposition 3.

Suppose that $(1 - \sigma)(A - \Lambda_0/M_0) > 0$ holds, and thus that there exists the optimal T^* such that $0 < T^* < \infty$. Also, suppose that the following approximations hold:

$$1 - \alpha\mu \approx 0 \quad \text{and} \quad 1 - \sigma \approx 0.$$

Then, the derivatives of $K_{T_-}^*$ and T^* with respect to η_h/η_0 have signs as follows:

$$\frac{dK_{T_-}^*}{d(\eta_h/\eta_0)} < 0, \quad \text{and} \quad (4.8)$$

$$\frac{dT^*}{d(\eta_h/\eta_0)} < 0 \quad \text{for } 1 > \sigma; \quad \frac{dT^*}{d(\eta_h/\eta_0)} > 0 \quad \text{for } 1 < \sigma. \quad (4.9)$$

(Proof is omitted.)

As defined in (2.5), η_h and η_0 are new and old coefficients, respectively, that links the growth rate of capital to that of environmental technology level. The proportion η_h/η_0 represents the degree of the innovation. Intuitively, it tells how much the transition to the new environmental technology is beneficial to the economy. Thus, it is easy to predict that a larger value of the proportion would make the optimal capital stock level for the transition smaller. (4.8) clarifies this point.

The indications of (4.9) are consistent to (4.8). When $1 > \sigma$, capital stock starts growing from a lower level and approaches to the level of $K_{T_-}^*$, which was shown by (4.6a) and in Figure 1. Thus, a smaller $K_{T_-}^*$ would make the time of the attainment earlier. To the contrary, when $1 < \sigma$, initial capital stock is sufficient to the economy, as shown by (4.6b), and no accumulation is needed. Declining capital stock eventually approaches the level of $K_{T_-}^*$ as was shown in Figure 2. Thus, a smaller $K_{T_-}^*$ would make the time of the attainment later.

These observations are depicted in Figures 3 and 4.

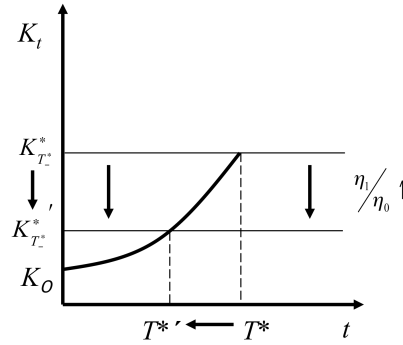


Figure 3. The impact of the change in η_h/η_0 for the case of $1 > \sigma$

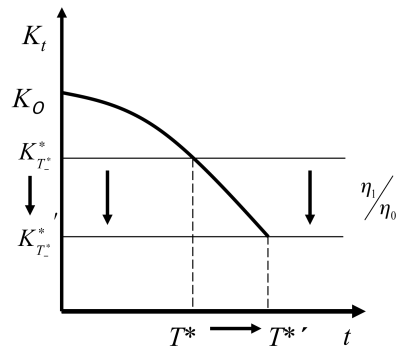


Figure 4. The impact of the change in η_h/η_0 for the case of $1 < \sigma$

5. Conclusion

This study investigated the optimal timing of transition in environmental technology from old one to new one along with economic growth. The economy enjoyed consumption as well as environmental quality, thus transition to an innovative environmental technology may benefit the economy while the transition may impose a huge cost to the economy. Consideration of net benefit yields the

optimal timing. The same idea has been proposed by Cunha-e-Sá and Reis (2007). We here developed a different model from theirs in that:

- we focused on acceleration (changes in the growth rate) of environmental technology, which is triggered by capital stock; and
- our treatment of the cost for the transition is rather simple and thus helps reduce ad hoc parameters representing costs in the model.

We obtained three propositions, which depict when as well as how transition of environmental technology occurs. The timing is endogenous and is characterized by the properties of the economies, in particular whether the economy is developing or developed. Findings are summarized as follows:

1. The primary determinant of whether transition to new technology is needed or not is the degree of complementarity between environmental quality and consumption. A secondary determinant is the average consumption propensity. The necessity of the transition implies either high degree of the complementarity with a low consumption propensity or negative complementarity (i.e. substitutability) with a high consumption propensity. (Proposition 1)
2. Given the necessity, transitions to new technology may occur in two possible ways. One is that an environmentally developing economy would accumulate social capital and invest it to the development of the new technology. The other way is that a matured economy holding a sufficiency of capital and enjoying a high consumption rate would realize the need for environmental quality improvement in some day. (Proposition 2)
3. The relation between the optimal timing of transition and the degree of the technology innovation is dependent upon the distinction of the above two cases. For the former case, a higher degree of innovation leads to earlier transition. However, for the latter case, the relation is the opposite: a higher degree of innovation makes the economy postpone the transition. (Proposition 3)

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